AP Calculus AB   Final Exam Review #1
Evaluate each limit.

1) $\lim\limits_{x \to 0} \dfrac{1 - \cos(4x)}{5x}$

2) $\lim\limits_{x \to 0} \dfrac{3x}{\tan(x)}$

3) $\lim\limits_{x \to 1} \dfrac{\sqrt{x+3} - 2}{x - 1}$

4) $\lim\limits_{x \to 0} \dfrac{\sin^2(3x)}{3x^2}$

5) $\lim\limits_{x \to -\infty} -\dfrac{20}{x^2 + 5}$

6) $\lim\limits_{x \to \infty} (x^5 - 2x^3 + x + 2)$

7) $\lim\limits_{x \to -3^+} f(x),\ f(x) = \begin{cases} -x^2 - 2x, & x \le -3 \\ 1, & x > -3 \end{cases}$

8) $\lim\limits_{x \to 0^+} f(x),\ f(x) = \begin{cases} x, & x < 0 \\ -2, & x \ge 0 \end{cases}$

9) $\lim\limits_{x \to 0} \dfrac{e^{4x} - 1}{2x}$

10) $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{2x^3}$

Use the 3-part definition of continuity to determine if each function is continuous. If the function is not continuous, find the *x*-axis location of and classify each discontinuity as removable or non-removable (infinite or jump).

11) $f(x) = \dfrac{x^2 - 1}{x + 1}$

12) $f(x) = \begin{cases} -2x - 3, & x < -2 \\ 2x + 5, & x \ge -2 \end{cases}$

13) $f(x) = -2x^2 - 4x - 1$

14) $f(x) = \begin{cases} x^2 - 4x + 4, & x < 1 \\ -2x + 7, & x \ge 1 \end{cases}$

Differentiate each function with respect to *x*.

15) $f(x) = 4x^4 + 3x^3 + 5x^{\frac{3}{2}}$

16) $f(x) = \dfrac{4x^2}{4x^{\frac{3}{4}} + 5}$

17) $f(x) = \sqrt[3]{4x^3 + 5}$

18) $f(x) = (x^4 + 2) \cdot e^{2x^5}$

19) $f(x) = \ln x^5$

20) $f(x) = \csc 5x^3$

21) $f(x) = \cot 2x^3$

22) $f(x) = \sec 4x^2$

23) $y = \sin^{-1} 5x^2$

24) $y = \tan^{-1} -x^5$

Answers to

1) 0

2) 3

3) $\dfrac{1}{4}$

4) 3

5) 0

6) ∞

7) 1

8) -2

9) 2

10) ∞

11) Removable discontinuity at: $x = -1$

12) Continuous

13) Continuous

14) Jump discontinuity at: $x = 1$

15) $f'(x) = 16x^3 + 9x^2 + \dfrac{15x^{\frac{1}{2}}}{2}$

16) $f'(x) = \dfrac{20x^{\frac{7}{4}} + 40x}{16x^{\frac{3}{2}} + 40x^{\frac{3}{4}} + 25}$

17) $f'(x) = \dfrac{4x^2}{\left(4x^3 + 5\right)^{\frac{2}{3}}}$

18) $f'(x) = \left(x^4 + 2\right) \cdot e^{2x^5} \cdot 10x^4 + e^{2x^5} \cdot 4x^3$
$= 2x^3 e^{2x^5}\left(5x^5 + 10x + 2\right)$

19) $f'(x) = \dfrac{1}{x^5} \cdot 5x^4$
$= \dfrac{5}{x}$

20) $f'(x) = -15x^2 \csc 5x^3 \cot 5x^3$

21) $f'(x) = -6x^2 \csc^2 2x^3$

22) $f'(x) = 8x \sec 4x^2 \tan 4x^2$

23) $\dfrac{dy}{dx} = \dfrac{10x}{\sqrt{1 - 25x^4}}$

24) $\dfrac{dy}{dx} = -\dfrac{5x^4}{x^{10} + 1}$